In-Plane Shearing Stress–Strain Response of Glass-Fiber-Reinforced Composites as Determined from Four-Point Bending Tests

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SYNOPSIS

In this article an experimental study to determine the longitudinal (or in-plane) shearing stress–strain response of a unidirectional fiber-reinforced composite material is presented. The test method used is the four-point bending of a ±45° off-axis glass-fiber-reinforced laminate. Although a laminate is used for the investigation of the shearing stress–strain response, it is shown that unidirectional shear properties can be found from the laminate test data following a procedure analogous to that used in previously. Also, the 45° off-axis test of the unidirectional composite in bending was carried out to obtain the in-plane shear modulus and compare it with that obtained by the ±45° off-axis method. Finally both values were compared with the theoretical value of the in-plane shear modulus obtained from a theoretical formula where the concept of boundary interphase between fiber and matrix was introduced. © 1995 John Wiley & Sons, Inc.

INTRODUCTION

The characterization of a fiber-composite material needs a great number of parameters. In particular, for a two-phase material the physical behavior of the system depends separately on the properties of the fiber and the matrix, as well as on the fiber–matrix interaction. A knowledge of shear properties is invaluable whenever the interfacial bonding or matrix failure of a composite is critical, such as in a composite structure subjected to compression loading.

A method for determining the in-plane shearing stress–strain response of a unidirectional composite follows. Let us consider the unidirectional fibrous composite material illustrated in Figure 1(a), where directions 1 and 2 are parallel and transverse to the fiber direction, respectively. If the material is under a plane stress state, four independent elastic constants are necessary for characterization in the principal material directions (1–2 axes). Ashton et al.[1] described these quantities in terms of elastic engineering constants as E_1, E_2, ν_{12}, or ν_{21} and G_{12}, where E_1 and E_2 are elastic moduli in directions 1 and 2, respectively; ν_{12} and ν_{21} are the major and minor Poisson's ratios, respectively; and G_{12} is the in-plane shear modulus. Methods for determining the first three constants are well established.[1] However, the in-plane or longitudinal shear stress–strain response of a unidirectional composite material is usually difficult to obtain.

There are several methods available in the literature to determine the shear response of composites, which are summarized by McKenna.[2] Chiao et al.[3] evaluated six methods of measuring the shear properties of fiber-reinforced composites. Adams and Walrath[4] modified the original Iosipescu shear test for measuring in-plane and interlaminar shear stiffness and strength of various types of composites. Ifju and Post[5] used a compact double notch specimen geometry for measuring in-plane shear testing. However, problems of stress concentration at the "notch tip" are always present in these specimens. Post et al.[6] performed an experimental analysis of micromechanical shear deformation of thick graphite/epoxy laminates by Moire interferometry under rail-shear loading conditions. Whitney and

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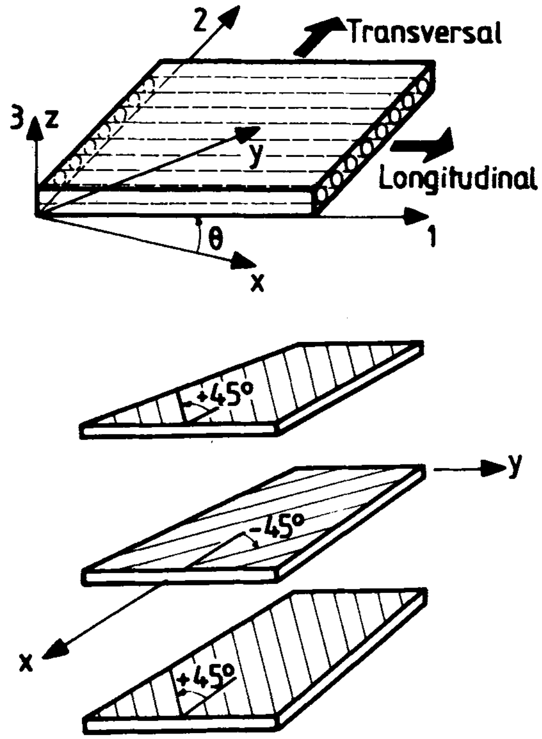

Figure 1 (a) Unidirectional fiber-reinforced material. (b) ±45° laminate.

Browning[7] critically examined the complex state of stresses for three-point and four-point short beam shear tests. Sims and Halpin[8] discussed the tensile shear of ±45° off-axis laminate and the rail shear test on a 0/90° laminate for determining the shear stress–strain response and concluded that these two methods have advantages over the torsion tube test.

The objective of this study is to: establish the four-point bend test of a ±45° laminate for the determination of the in-plane shear stress–strain response; and compare the results with those obtained from the 45° off-axis four-point bending test of a unidirectional fiber composite with the theoretical values of the in-plane shear modulus obtained from a theoretical formula where the concept of boundary interphase between fiber and matrix is used.

THEORETICAL ANALYSIS

By this analysis it is shown that the use of a ±45° laminate under four-point bending can predict the in-plane shearing stress–strain response of a unidirectional fiber reinforced composite material. The method of data reduction in the ±45° laminate test can be simplified so that the shearing stress–strain response can be determined without a knowledge of E_1, E_2, and ν_{12}. This is true because there is no coupling between shear and normal stress–strain behavior in the ±45° laminate.

Let us consider the ±45° laminate illustrated in Figure 1(b) where 1 and 2 are principal material axes and x, y are arbitrary laminate axes.

The constitutive equations for a laminate subjected to bending moments can be written as[9]:

$$\begin{Bmatrix} M_x \\ M_y \\ M_{xy} \end{Bmatrix} = \begin{bmatrix} B_{11} & B_{12} & B_{16} \\ B_{12} & B_{22} & B_{26} \\ B_{16} & B_{26} & B_{66} \end{bmatrix} \begin{Bmatrix} \varepsilon_x^0 \\ \varepsilon_y^0 \\ \gamma_{xy}^0 \end{Bmatrix} + \begin{bmatrix} D_{11} & D_{12} & D_{16} \\ D_{12} & D_{22} & D_{26} \\ D_{16} & D_{26} & D_{66} \end{bmatrix} \begin{Bmatrix} \kappa_x \\ \kappa_y \\ \kappa_{xy} \end{Bmatrix} \quad (1)$$

where ε_x^0, ε_y^0, γ_{xy}^0 are the middle surface strains; κ_x, κ_y, κ_{xy} are the middle with surface curvatures; and M_x, M_y, M_{xy} the applied moments per unit width.

$$D_{ij} = \tfrac{1}{3} \sum_{k=1}^{N} (\bar{Q}_{ij})_k (z_k^3 - z_{k-1}^3) \quad (2)$$

$$B_{ij} = \tfrac{1}{2} \sum_{k=1}^{N} (\bar{Q}_{ij})_k (z_k^2 - z_{k-1}^2) \quad (3)$$

where D_{ij} are the elements of the laminate bending stiffness matrix and B_{ij} the elements of the laminate coupling stiffness matrix. The presence of the B_{ij} implies coupling between bending and extension of a laminate. For a symmetric laminate:

$$B_{ij} = 0 \rightarrow \{M\} = [\mathbf{D}]\{\kappa\}$$

Equation (2) can be written as

$$D_{ij} = \sum_{k=1}^{N} (\bar{Q}_{ij})_k \left(t_k \bar{z}_k^2 + \frac{t_k^3}{12} \right) \quad (4)$$

where t_k is the thickness and $\bar{z}_k$ is the distance to the centroid of the kth layer. In this relationship $\bar{Q}_{ij}$ are the components of the reduced stiffness matrix.[9]

For a ±45° symmetric laminate we find

$$(\bar{Q}_{11})_{45} = \tfrac{1}{4}[Q_{11} + Q_{22} + 2Q_{12} + 4Q_{66}] = (\bar{Q}_{11})_{-45}$$

$$(\bar{Q}_{22})_{45} = \tfrac{1}{4}[Q_{11} + Q_{22} + 2Q_{12} + 4Q_{66}] = (\bar{Q}_{22})_{-45}$$

$$(\bar{Q}_{12})_{45} = \tfrac{1}{4}[Q_{11} + Q_{22} + 2Q_{12} - 4Q_{66}] = (\bar{Q}_{12})_{-45}$$

$$(\bar{Q}_{66})_{45} = \tfrac{1}{4}[Q_{11} + Q_{22} - 2Q_{12}] = (\bar{Q}_{66})_{-45}$$

$$(\bar{Q}_{16})_{45} = (\bar{Q}_{16})_{-45} = (\bar{Q}_{26})_{45} = (\bar{Q}_{26})_{-45} = 0 \qquad (5)$$

with

$$Q_{11} = \frac{E_1}{1 - \nu_{12}\nu_{21}}, \quad Q_{12} = \frac{\nu_{12}E_2}{1 - \nu_{12}\nu_{21}} = \frac{\nu_{21}E_1}{1 - \nu_{12}\nu_{21}}$$

$$Q_{22} = \frac{E_2}{1 - \nu_{12}\nu_{21}}, \quad Q_{66} = G_{12}. \qquad (6)$$

For the 13 layer ±45° symmetric laminate, used in this study, the elements of [**D**] matrix are calculated as:

$$\begin{aligned} D_{ij} &= (\bar{Q}_{ij})_{45}t_k\left[(6t_k)^2 + \frac{t_k^2}{12}\right] + (\bar{Q}_{ij})_{-45}t_k\left[(5t_k)^2 + \frac{t_k^2}{12}\right] \\ &+ (\bar{Q}_{ij})_{45}t_k\left[(4t_k)^2 + \frac{t_k^2}{12}\right] + (\bar{Q}_{ij})_{-45}t_k\left[(3t_k)^2 + \frac{t_k^2}{12}\right] \\ &+ (\bar{Q}_{ij})_{+45}t_k\left[(2t_k)^2 + \frac{t_k^2}{12}\right] + (\bar{Q}_{ij})_{-45}t_k\left[(t_k^2 + \frac{t_k^2}{12}\right] \\ &+ (\bar{Q}_{ij})_{45}t_k\frac{t_k^2}{12} + (\bar{Q}_{ij})_{-45}t_k\left[(-t_k)^2 + \frac{t_k^2}{12}\right] \\ &+ (\bar{Q}_{ij})_{45}t_k\left[(-2t_k)^2 + \frac{t_k^2}{12}\right] + \cdots \\ &= 2197\frac{t_k^3}{12}(\bar{Q}_{ij})_{45} = \frac{t^3}{12}(\bar{Q}_{ij})_{45} \end{aligned} \qquad (7)$$

where t denotes the total laminate thickness.

Substituting Eqs. (5) and (7) in Eq. (1) we find

$$\begin{Bmatrix} M_x \\ M_y \\ M_{xy} \end{Bmatrix} = \frac{t^3}{12}\begin{bmatrix} \frac{1}{4}(Q_{11} + Q_{22} + 2Q_{12} + 4Q_{66}) & \frac{1}{4}(Q_{11} + Q_{22} + 2Q_{16} - 4Q_{56}) & 0 \\ \frac{1}{4}(Q_{11} + Q_{22} + 2Q_{12} - 4Q_{66}) & \frac{1}{4}(Q_{11} + Q_{22} + 2Q_{12} + 4Q_{66}) & 0 \\ 0 & 0 & \frac{1}{4}(Q_{11} + Q_{22} - 2Q_{12}) \end{bmatrix}\begin{Bmatrix} \kappa_x \\ \kappa_y \\ \kappa_{xy} \end{Bmatrix}. \qquad (8)$$

If the state of applied moment is: $M_x = M$ and $M_y = M_{xy} = 0$ we have

$$M = \frac{t^3}{48}[(Q_{11} + Q_{22} + 2Q_{12})(\kappa_x + \kappa_y) + 4Q_{66}(\kappa_x - \kappa_y)] \qquad (9a)$$

$$0 = \frac{t^3}{48}[(Q_{11} + Q_{22} + 2Q_{12})(\kappa_x + \kappa_y) - 4Q_{66}(\kappa_x - \kappa_y)] \qquad (9a, b)$$

$$0 = \frac{t^3}{48}[(Q_{11} + Q_{22} - 2Q_{12})\kappa_{xy}] \qquad (9c)$$

Substracting the second relationship from the first one we find

$$M = 6Q_{66}t^3(\kappa_x - \kappa_y) \qquad (10)$$

Because $Q_{66} = G_{12}$ we obtain the final relationship

$$G_{12} = \frac{6M}{(\kappa_x - \kappa_y)t^3}. \qquad (11)$$

Because of a possible asymmetry in the specimen (nonuniform thickness, existence of resin-rich layer) or because in composite materials the tensile and the compressive elastic modulus are usually not equal, when the beam is subjected to four-point bending the neutral axis is not at the middle of the thickness, t. In this case, the curvatures in x and y directions are measured by two sets of strain gauges, stuck on the upper and the lower surfaces of the beam at the same cross sections. Then κ_x and κ_y are given as

$$\kappa_x = (\varepsilon_{x_1} - \varepsilon_{x_2})/t \qquad (12a)$$

$$\kappa_y = (\varepsilon_{y_1} - \varepsilon_{y_2})/t. \qquad (12b)$$

Substituting the above relationships in Eq. (11) and with $M_x = M = Pl/3b$ where l is the span length and t the specimen thickness we have

$$G_{12} = \frac{2PL}{t^3b[(\varepsilon_{x_1} - \varepsilon_{x_2}) - (\varepsilon_{y_1} - \varepsilon_{y_2})]}. \qquad (13)$$

If there is no asymmetry in the specimen or when the composite elastic moduli in tension and under compression are equal, then $\varepsilon_{x_2} = -\varepsilon_{x_1}$ and $\varepsilon_{y_2} = -\varepsilon_{y_1}$. Thus

$$G_{12} = \frac{PL}{t^3b(\varepsilon_{x_1} - \varepsilon_{y_1})}. \qquad (14)$$

From the above equation the shear stress–strain relationship can be obtained. Indeed, the bending stress, σ_x, and the normal strains, ε_x and ε_y, in the x and y directions are related to the shear stress τ_{12} and shear strain γ_{12} in the principal material axes, 1 and 2, respectively with the aid of the following formulae:

$$\tau_{12} = -\frac{\sigma_x + \sigma_y}{2}\sin 2\theta + \frac{\sigma_x - \sigma_y}{2}\cos 2\theta + \tau_{xy}\sin 2\theta \quad (15)$$

$$\gamma_{12} = (\varepsilon_y - \varepsilon_x)\sin 2\theta + \gamma_{xy}\cos 2\theta. \quad (16)$$

The above relationships for $\sigma_y = \tau_{xy} = 0$ and $\theta = -45°$ yield:

$$\tau_{12} = \sigma_x/2 \quad \text{and} \quad \gamma_{12} = \varepsilon_x - \varepsilon_y. \quad (17a,b)$$

Consequently

$$G_{12} = \frac{\tau_{12}}{\gamma_{12}} = \frac{\sigma_x}{2(\varepsilon_x - \varepsilon_y)}. \quad (18)$$

This relationship is the same with Eq. (14) because

$$\sigma_x = \frac{6M_x}{t^2} = \frac{6PL}{3bt^2}.$$

The normal strains are measured by strain gauges for increasing values of bending loads and thus the shear stress–strain diagram can be traced. It is concluded that the unidirectional shearing stress–strain response can be determined from the laminate normal stress–strain response through a bending experiment.

Also, the 45° off-axis bending test of a unidirectional fiber composite was performed to determine the in-plane shear modulus according to the following expression[9]

$$G_{12} = \frac{1}{\left(\frac{4}{E_{45}} - \frac{1}{E_1} - \frac{1}{E_2} + \frac{2\nu_{12}}{E_1}\right)}. \quad (19)$$

The experimental results were compared with the theoretical value of the in-plane shear modulus obtained from an expression derived in a previous work using the concept of boundary interphase.[10]

According to this model whose representative volume element (RVE) for a fiber reinforced composite material appears in Figure 2, the composite consists of three different materials. The fiber is surrounded by the interphase and this in turn is surrounded by the matrix.

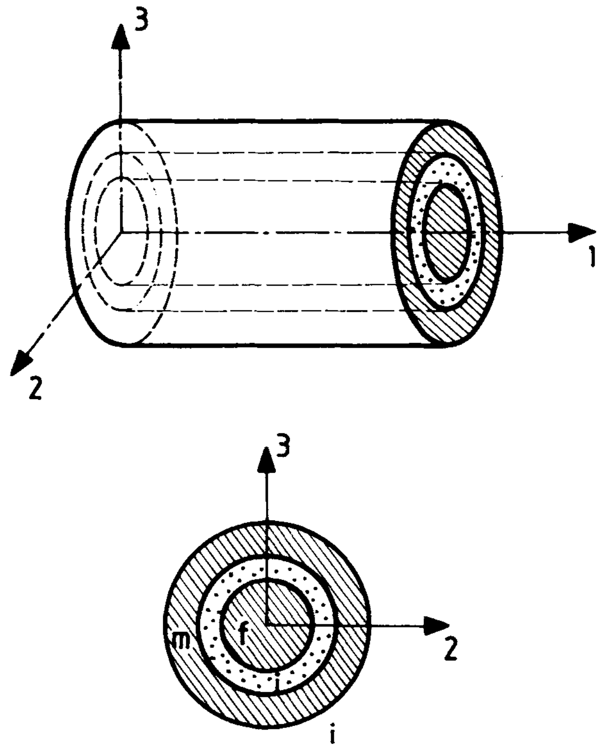

Figure 2 Representative volume element of the unidirectional fiber composite including the boundary interphase.

If we denote by r_f, r_i, r_m the outer radii of the fiber, the interphase and the matrix, respectively, then the volume fractions of the respective phases are given by:

$$u_f = \frac{r_f^2}{r_m^2}, \quad u_i = \frac{r_i^2 - r_f^2}{r_m^2}, \quad u_m = \frac{r_m^2 - r_i^2}{r_m^2} \quad (20)$$

with $u_f + u_i + u_m = 1$.

The expression for the in-plane shear modulus, G_{12}, is given as follows[10]:

$$G_{12} = \frac{G_m\left\{4G_m\left[G_f u_f + \frac{2u_f}{r_f^2}\int_{r_f}^{r_i} G_i(r) r\, dr\right] + [(G_i^2 + G_m^2)(2 - u_m) + 2G_iG_m u_m]u_m\right\}}{[G_i u_m + G_m(2 - u_m)]^2} \quad (21)$$

where

$$G_i = \frac{1}{u_i}\int_{r_f}^{r_i} G_i(r)\, du.$$

In the above expression G_f, G_i, G_m denote the shear moduli and u_f, u_i, u_m the volume fraction of fiber, interphase, and matrix, respectively. The interphase shear modulus $G_i(r)$ was determined assuming a polynomial variation (linear, hyperbolic, parabolic, etc.)

$$G_i(r) = Ar^n + Br^{n-1} + Cr^{n-2} + \cdots.$$

with the following boundary conditions:

$$G_i(r_f) = G_f, \quad G_i(r_i) = G_m, \quad \text{and} \quad \frac{dG_i(r_i)}{dr}. \tag{22}$$

This assumption is based on the fact that the interphase constitutes a transition zone between fibers, usually with high moduli, and the matrix with rather low moduli.

The interphase thickness and volume fraction were determined according to a procedure described by Lipatov.[11] For $u_f = 0.65$ the interphase volume fraction and shear modulus were found[10] as

$$u_i = 0.051 \quad \text{and} \quad G_i = 15.56 \times 10^9\ \text{N/m}^2$$

respectively, as the in-plane shear modulus of the composite was evaluated as $G_{12} = 6.45 \times 10^9$ N/m^2.

The elastic constants of the fiber and matrix material used in the theoretical calculations are as follows:

$$E_f = 72\ \text{GPa}, \quad E_m = 3.5\ \text{GPa},$$
$$\nu_f = 0.20, \quad \text{and} \quad \nu_m = 0.35.$$

EXPERIMENTAL

Two groups of specimens were tested in the experiments. Both the 13 layer symmetric ±45° laminate and the unidirectional glass-fiber composite used in the present investigation consisted of an epoxy matrix reinforced with long E-glass fibers (Permaglass XE5/1, Permali Ltd, U.K.). The matrix material was based on a diglycidyl ether of bisphenol A together with an aromatic amine hardener (Araldite M_y 750/HT972, Ciba-Geigy, U.K.). The glass fibers had a diameter of 12×10^{-6} m and were contained at a volume fraction $u_f = 0.65$.

The volume fraction u_f was determined according to British Standards by igniting samples of the composite and weighing the residue, which gave the weight fraction of glass as 79.6 ± 0.28%. This and the measured values of the relative densities of permaglass ($\rho_f = 2.55$ gr/cm^3) and the epoxy matrix ($\rho_m = 1.2$ gr/cm^3) gave the value $u_f = 0.65$. More details about the material and the experiments of thermal analysis can be found elsewhere.[10]

The dimensions of the specimens and the conditions of the bending experiments were determined according to ASTM D-790 for the four-point bend-

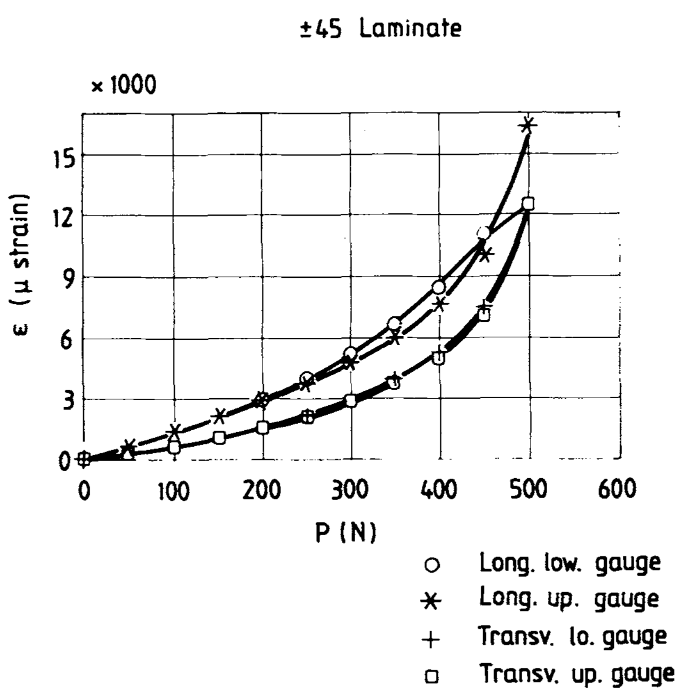

Figure 3 Load-strain diagram for the ±45° laminate in bending.

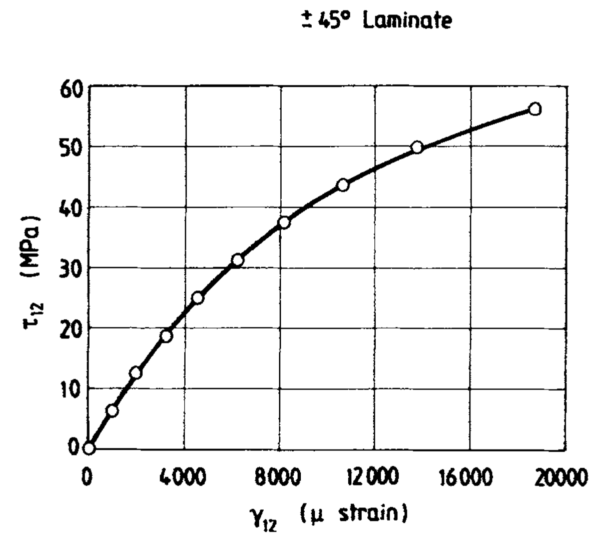

Figure 4 In-plane shear stress–strain response for the ±45° laminate in bending.

ing method. Given that the thickness of the plate was 6–7 mm, the dimensions were chosen as L (total length) = 253 mm, b (width) = 20 mm, l (span length) = 203 mm, l' (length between applied loads) = 68 mm with l/t = 32/1.

Strain gauges (type KYOWA KCP-2C1-65, gauge length 2 mm and gauge factor k = 1.99) were bonded in the central part of the specimen at 0° and 90° in order to measure the respective strains. The specimens were tested in bending at a crosshead speed of 0.2 cm/min.

RESULTS

Figure 3 presents the load-strain diagram for glass–epoxy ±45° laminate in bending. From this experiment using Eq. (13) and with the previously mentioned specimen dimensions [b = 19.8 mm, t = 6.3 mm, l = 203 mm] the value $G_{12} = 6.51 \times 10^9$ N/m^2 can easily be found.

The in-plane shear stress–strain response for the glass–epoxy ±45° laminate as calculated from Eq. (17a,b) is illustrated in Figure 4. It can be observed that there is a good agreement between the experimentally obtained in-plane shear modulus G_{12} = 6.51 GPa and the theoretical one,[10] G_{12} = 6.45 GPa, where the concept of boundary interphase was used.[10]

Figure 5 presents the load-strain diagram for glass–epoxy (u_f = 0.65) 45° unidirectional composite in bending. From this experiment with the aid of Eq. (19) and by using the experimental values of the elastic constants E_1, E_2, ν_{12} obtained by Kyriazi and Sideridis[12] the in-plane shear modulus is found as $G_{12} = 5.87 \times 10^9$. Thus, it can be observed that there is a discrepancy (9.9%) between this value and the experimental value of G_{12} obtained from the ±45° laminate.

CONCLUSIONS

An experimental study has been presented for determining the in-plane shearing stress–strain re-

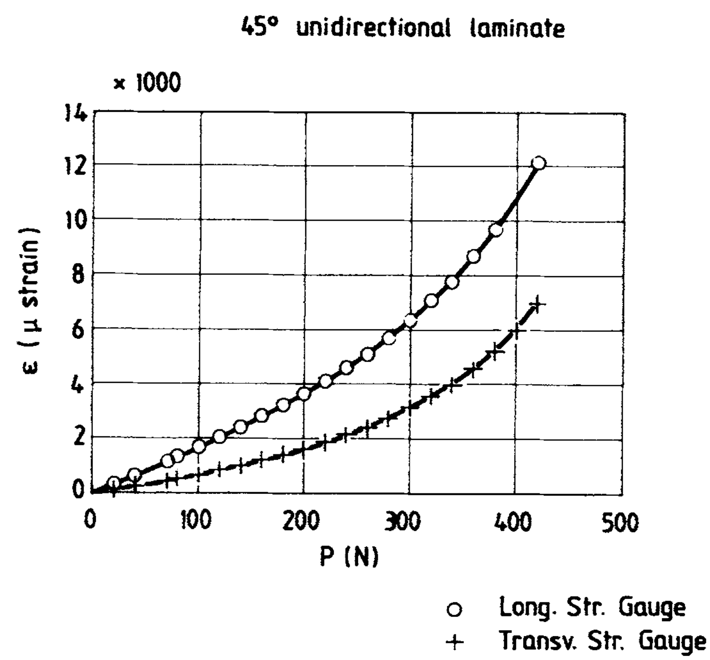

Figure 5 Load-strain diagram for the 45° unidirectional laminate in bending.

sponse of glass-fiber–epoxy composites from the stress–strain results of a four-point bend test on a ±45° laminate and a four-point bend test on a +45° off-axis laminate. It has been shown that there is little discrepancy between the values of shear modulus obtained by these two methods, the main advantages of which are that test specimens are inexpensive and easy to fabricate, the tests are easy to carry out, and data reduction is relatively simple.

The concept of the boundary interphase between fiber and matrix introduced in a previous article was used in the derivation of the theoretical expression for the shear modulus. The shear data obtained from theory was compared with that obtained from each of the two test methods. The comparison has shown that the theoretical result was closer to the experimental one obtained from the bend test of the ±45° laminate.

REFERENCES

1. J. E. Ashton, J. C. Halpin, and P. H. Petit, *Primer on Composite Materials: Analysis,* Technomic, Stamford, CT, 1969.
2. G. B. McKenna, *Interlaminar Effects in Fibre-Reinforced Plastics,* Marcel Dekker Inc., New York, 1975.
3. C. C. Chiao, R. L. Moore, and T. T. Chiao, *Composites,* **9,** 161 (1977).
4. D. F. Adams and D. E. Walrath, *Exp. Mech.,* **27,** 113 (1987).
5. P. Ifju and D. Post, Proc. SEM Spring Conf. Exp. Mech., Boston MA, 1989, p. 337.
6. D. Post, F. Dai, Y. Guo, and P. Ifju, *J. Comp. Mater.,* **23,** 264 (1989).
7. J. M. Whitney and C. E. Browning, *Exp. Mech.,* **25,** 294 (1985).
8. D. F. Sims and J. C. Halpin, Compos. Mater.: Testing Design (Third Conference), ASTM STP 546, American Society for Testing and Materials, Philadelphia, (1974), p. 46.
9. R. M. Jones, *Mechanics of Composite Materials,* McGraw–Hill, Tokyo, 1975.
10. E. Sideridis, *Comp. Sci. Technol.,* **31,** 35 (1988).
11. Y. S. Lipatov, *Physical Chemistry of Filled Polymers,* originally published *Khimiya,* Moscow, 1977. Translated from Russian by R. J. Moseley, *Int. Polym. Sci. Technol.,* Monograph No. 2.
12. E. Kyriazi and E. Sideridis, to appear.

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